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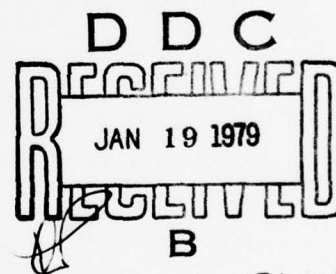
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APPROXIMATE FINITE-ELEMENT METHOD OF STRESS ANALYSIS OF NON-AXISYMMETRIC CONFIGURATIONS

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Stress Analysis   Non-Axisymmetric Configurations
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A finite element method of analysis is developed for structural configurations which are derived from axisymmetric geometries but contain definite nonaxisymmetric features in the circumferential direction. The purpose of the present analysis is to develop a method which will take into consideration the fact that the stress and strain conditions in these geometries will be related to the corresponding axisymmetric solution. To analyze these structures the geometry is divided into several segments in the r-θ plane. The

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axisymmetric displacements are obtained for each segment by solving a related axisymmetric configuration. A perturbation analysis is then performed to match the solutions at certain points between the segments, and obtain the perturbation displacements for the total structure. The total displacement is then the axisymmetric displacement plus the perturbation displacement. The stresses and strains are then calculated at any desired point once the total displacements are known. The method is applied to a number of examples to illustrate the accuracy of the method. The results for these examples are presented and discussed.

I. INTRODUCTION

Many problems in structural analysis have an axisymmetric configuration. However, there are other problems which, while they are almost axisymmetric, have non-symmetric features which render purely axisymmetric solutions inapplicable. One approach to solving these types of problems would be to use a three-dimensional finite element formulation. However, if some advantage can be taken of the fact that these geometries will yield solutions somewhat similar to the axisymmetric cases, some efficiency could be gained over the purely three-dimensional approach. Consequently, a method that makes use of two-dimensional axisymmetric analysis, but also yields first order non-symmetric effects, is an appealing alternative.

This paper attempts to develop an analysis of the stress and displacement field of slightly non-axisymmetric bodies. This is done by obtaining the axisymmetric displacements for various parts of the body, and then forcing the displacements to match at certain points through a perturbation analysis. The axisymmetric displacements together with the perturbation displacements combine to yield the total displacements for each section of the body. Once the displacements are known, it is a simple matter to calculate stresses and strains at any desired point in the structure.

II. METHOD OF ANALYSIS

The purpose of the present investigation is to develop an approximate method of analysis of elastic configurations whose geometry is almost axisymmetric, but has pronounced geometrical changes in the circumferential direction. The method of analysis which is proposed is based on the finite element numerical approach, and will attempt to make some use of the geometrical aspects of the two-dimensional axisymmetric analysis.[1], [2]* Although it is not necessary for the proposed method, let us for the purpose of discussion assume that the geometry repeats at some regular intervals in the circumferential direction and therefore, only one portion of the total structure need be considered. Of course, if there is no repetitions then the total structure would be analyzed. Consider now one such portion as illustrated in Figure 1. For the purpose of analysis the configuration shown in Figure 1 is divided into a set of finite elements whose quadrilateral shape is defined in the r-z plane.[2] At this point direct use can be made of a suitable mesh generation procedure in a two-dimensional plane.[1] On each face of this segment the finite element nodes are assumed to correspond to the nodes in the adjacent segments.

The first step is to obtain the axisymmetric solution for each of these segments. In addition, the stiffness and force matrices for the non-axisymmetric solution are calculated and stored. In general, the displacements generated by the axisymmetric solution will not be compatible with the displacements generated for an adjacent segment. Consequently, some changes need to be made in these displacements to obtain the real situation. The total displacement is now represented by two components. If we call the displacement u we can write:

$$\{u\} = \{u_a\} + \{u_p\} \tag{1}$$

where $\{u_a\}$ is the displacement from the axisymmetric solution and $\{u_p\}$ is the additional perturbation displacement.

For each side of a segment the perturbation displacements are expressed as follows:

$$u_r = a_1 + a_2 r + z(a_3 + a_4 r)$$

$$u_z = a_5 + a_6 r + z(a_7 + a_8 r) \tag{2}$$

$$u_\theta = a_9 + a_{10} r + z(a_{11} + a_{12} r)$$

where subscripts r, z, θ refer to coordinates in Figure 1.

* Superscripts refer to references

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Similar expressions can be written for the other side of the segment.

If the total displacements are forced to match for two adjacent segments at four nodal points, a sufficient number of equations to be able to solve for the coefficients of the perturbation displacement equations will be obtained. These nodes, where the solutions match are called connecting nodes. Then for each segment:

$$\{u_{c_p}\} = [R] \{p\} \tag{3}$$

where $\{u_{c_p}\}$ is the perturbation displacement at the connecting nodes, $[R]$ is matrix of the connecting nodal coordinates, and $\{p\}$ is the unknown perturbation coefficients. Solving for $\{p\}$ yields:

$$\{p\} = [R]^{-1} \{u_{c_p}\} \tag{4}$$

Also, for each quadrilateral element in the segment:

$$\{u_p\} = [G] \{p\} \tag{5}$$

where $[G]$ is the matrix of nodal coordinates for the element. Substituting equation (4) into equation (5) yields:

$$\{u_p\} = [G] [R]^{-1} \{u_{c_p}\} \tag{6}$$

Also for each element:

$$\{f\} = [S_1] (\{u_a\} + \{u_p\}) \tag{7}$$

Where $[S_1]$ is the non-axisymmetric stiffness matrix and $\{f\}$ is the internal force vector. Now, applying the principal of virtual work to equation (7) gives:

$$\delta u_p^T \{f\} = \delta u_p^T [S_1](\{u_a\} + \{u_p\}) \tag{8}$$

From equation (6), it can be seen that:

$$\delta u_p^T = \delta u_{c_p}^T [R]^{-1^T} [G]^T \tag{9}$$

Substituting this into equation (8) and summing the contribution of all the *elements in the segment*, defines the following:

$$W_I = \delta u_{c_p}^T \sum ([R]^{-1^T} [G]^T [S_1] \{u_a\} + [R]^{-1^T} [G]^T [S_1] [R]^{-1} \{u_{c_p}\}) \tag{10}$$

Also, there is contributions from the external forces. This can be written as:

$$\delta u_p^T \{f_E\} = \delta u_p^T \{P_1\} \tag{11}$$

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where $\{P_1\}$ is the external force vector for an individual element. The same process of substitution and summing over all the elements defines the following:

$$W_E = \delta u_{c_p}^{\ T} \sum [R]^{-1^T} [G]^T \{P_1\} \tag{12}$$

Also there will be a contribution from the concentrated loads. This contribution can be stated as:

$$\delta u_p^{\ T} \{f_c\} = \delta_{u_p}^{\ T} \{g\} \tag{13}$$

where $\{g\}$ is the concentrated force vector for the element. Then, for the whole segment:

$$W_C = \delta u_{c_p}^{\ T} \sum [R]^{-1^T} [A]^T \{g\} \tag{14}$$

where $[A]$ is a matrix relating the perturbation displacements at the nodes where the concentrated forces are applied to the perturbation coefficient matrix. That is:

$$\{u_p\} = [A] \{p\} \tag{15}$$

at nodes where concentrated forces are applied.

Balancing the virtual work done by all the forces gives:

$$W_E + W_C = W_I \tag{16}$$

Here, the following substitutions can be made:

$$F_E = [R]^{-1^T} \left(\sum [G]^T \{P_1\} + [A]^T \{g\} \right)$$

$$[S_K] = [R]^{-1^T} \left(\sum [G]^T [S_1] [G] \right) [R]^{-1} \tag{17}$$

$$\{F_I\} = [R]^{-1^T} \left(\sum [G]^T [S_1] \{u_a\} \right)$$

where the $[R]^{-1^T}$ has been brought outside the summation since it is constant for the entire segment, and the summation is over all the elements in the segment. The terms in the summation can be easily calculated during the axisymmetric solution and later be multiplied by the $[R]^{-1}$ terms. Then $\{F_E\}$, $[S_K]$, and $\{F_I\}$ can be stored for each segment. These substitutions give:

$$[S_K] \{u_{c_p}\} + \{F_I\} = \{F_E\} \tag{18}$$

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Since it is the total displacements that must match at the connecting nodes, equation (18) can be written as:

$$[S_K] \{u_{c_T}\} - [S_K]\{u_{c_a}\} + \{F_I\} = \{F_E\} \qquad (19)$$

where $\{u_{c_a}\}$ is the axisymmetric displacement at the connecting nodes and $\{u_{c_T}\}$ is the total displacement at the connecting nodes. Everything is known except $\{u_{c_T}\}$. Combining terms in equation (19) yields:

$$[S_K] \{u_{c_T}\} = \{F_C\} \qquad (20)$$

where $\{F_C\}$ is a combination of $\{F_E\}$, $\{F_I\}$, and $[S_K] \{u_{c_a}\}$. There will be one matrix equation for each segment. Now $[S_K]$ can be assembled into a banded global stiffness matrix for the overall structure. Then appropriate boundary conditions can be applied to restrict the rigid body motion, preserve symmetries, or comply with external restraints. Then equation (20) can be solved for the $\{u_{c_T}\}$ vector, giving the total displacements at all of the connecting nodes.

Now it is a simple matter to work back from this point to obtain the stresses and strains for each element in each segment. For a given segment $\{u_{c_T}\}$ and $\{u_{c_a}\}$ are known and therefore, $\{u_{c_p}\}$ can be calculated from:

$$\{u_{c_p}\} = \{u_{c_T}\} - \{u_{c_a}\} \qquad (21)$$

Then knowing $\{u_{c_p}\}$ and $[R]^{-1}$, $\{p\}$ can be obtained from equation (4). Then for each element the perturbation displacements $\{u_p\}$ can be obtained from equation (5). Then knowing $\{u_a\}$ and $\{u_p\}$, it is easy to get $\{u\}$ from equation (1).

At this point, simply apply the base finite element equations:

$$\{\varepsilon\} = [B] \{u\}$$
$$\{\sigma\} = [D] \{\varepsilon\} \qquad (22)$$

where $[B]$ and $[D]$ were calculated and saved from the axisymmetric solution, to obtain the stresses and strains for each element.

III. NUMERICAL RESULTS

The first test of this method is that if the body is indeed completely axisymmetric, the perturbation analysis should give the original axisymmetric answers. This was tested on two examples; a spinning disk with a hole in the center, and a disk under internal pressure. In both cases the axisymmetric solution was duplicated.

The next example was a non-axisymmetric disk under internal pressure. The answers for this example were compared to answers obtained from a plane stress analysis. This example is basically a plane stress problem in the

r-θ plane. The configuration that was analyzed is shown in Figure 3. Due to symmetry only one quarter of the total body was divided into six segments for the analysis, numbered as shown in Figure 3. The predominant stress for this example is $\sigma_{\theta\theta}$. Results of the perturbation analysis, plane-stress analysis and the axisymmetric analysis for all segments are shown in Table 1. It can be seen that the perturbation analysis gives close agreement with the plane stress solution.

The next example tested was a tube that was axisymmetric for half its length and had a cross-section as shown in Figure 3 for the other half of its length. The only test of accuracy that could be made on this example was to see if the answers tended to the known limit solutions at each end. That is that near the end of the tube the answers for the nonaxisymmetric part tended to those presented in Table 1, and at the other end the answers tended toward the axisymmetric solution. These results for all segments are presented in Table 2. From this we can see that the stresses vary from end to end and that they are approaching the limiting values for the case of the tube having no variation in the z direction.

IV. CONCLUSION

From the test examples used, it can be concluded that this method of solution can handle non-axisymmetric geometries. Also, the accuracy seems to be sufficient for bodies that have geometry varying in the r-θ plane, the r-z plane or both. Therefore, this method presents a simpler alternative to a purely three dimensional analysis, and for a large class of problems this would appear to have a great advantage in terms of saving in the computer execution time.

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REFERENCES

1. A. R. Zak, "Numerical Analysis of Laminated Orthotropic Composite
    Structures," TR AAE 75-2, University of Illinois, March 1975.

2. O. C. Zienkieviz, "The Finite Element Method in Engineering Science,"
    McGraw-Hill, London, (1971).

TABLE 1. Stress $\sigma_{\theta\theta}$ from various methods of analysis of a disk with internal pressure; p = 1000 Pa.

STRESS $\sigma_{\theta\theta}$

Radial Distance	Perturbation	Plane Stress	Axisymmetric	
6.25	1164	1216	857	
8.75	505	540	491	Segment 1
11.25	208	215	345	
13.75	46	20	271	
6.25	1168	1205	857	
8.75	505	558	491	Segment 2
11.25	207	217	345	
13.75	45	11	271	
6.25	1175	1178	857	
8.75	507	612	491	Segment 3
11.25	206	209	345	
13.75	42	8	271	
6.25	1065	1130	1198	
8.75	916	868	764	Segment 4
6.25	1074	1133	1198	
8.75	909	865	764	Segment 5
6.25	1078	1133	1198	
8.75	906	864	764	Segment 6

TABLE 2. Stress $\sigma_{\theta\theta}$ from perturbation analysis for a nonaxisymmetric under internal pressure ; p = 1000 Pa.

STRESS $\sigma_{\theta\theta}$

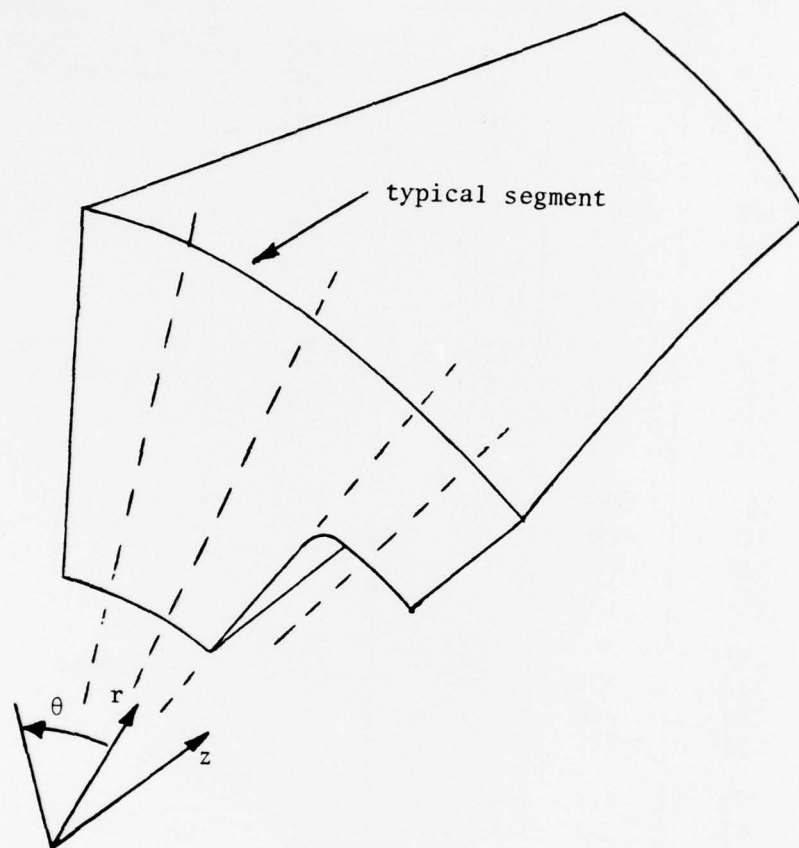| Radial Distance	Axisymmetric end		Nonaxisymmetric end		
	Tube	Disk	Tube	Disk	
6.25	934	857	1010	1164	
8.75	486	491	503	505	Segment 1
11.25	301	345	284	208	
13.75	205	271	108	46	
6.25	909	857	1043	1164	
8.75	484	491	505	505	Segment 2
11.25	314	345	270	208	
13.75	229	271	143	46	
6.25	847	857	1105	1164	
8.75	480	491	511	505	Segment 3
11.25	339	345	244	208	
13.75	273	271	97	46	
6.25	799	857	1067	1078	
8.75	456	491	878	906	Segment 4
11.25	328	345			
13.75	265	271			
6.25	757	857	1127	1078	
8.75	485	491	832	906	Segment 5
11.25	394	345			
13.75	354	271			
6.25	755	857	1155	1078	
8.75	479	491	811	906	Segment 6
11.25	386	345			
13.75	345	271			

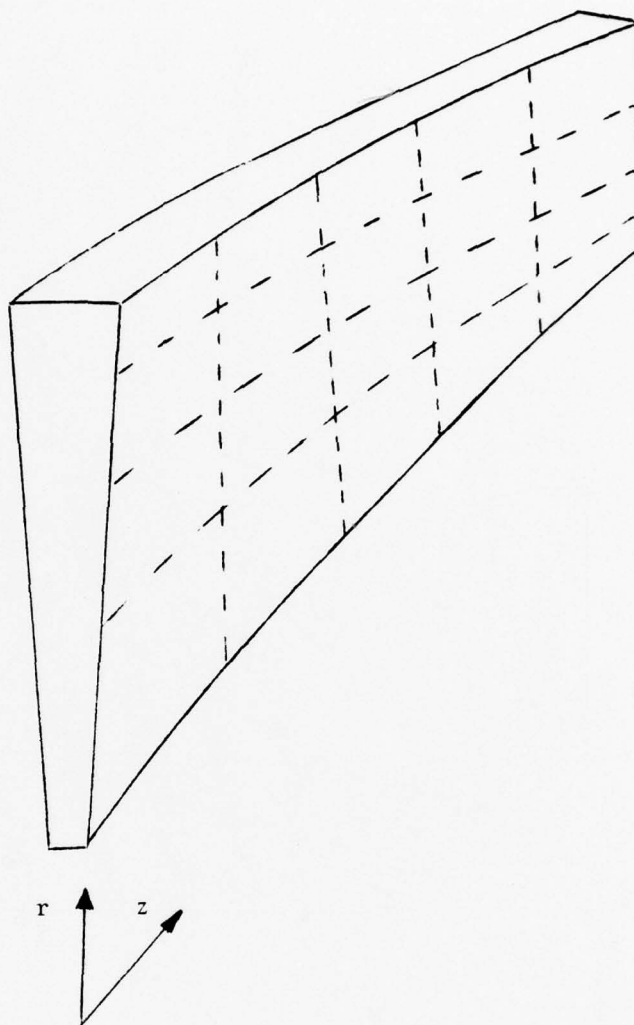Figure 1. Configuration to be analyzed is divided into segments by plane sections in the r-z plane.

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Figure 2.   Division of a typical segment into a finite-element grid.

13
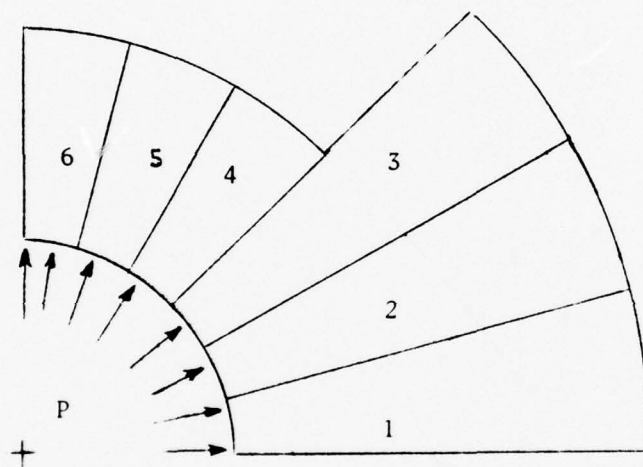
Figure 3. End view of nonaxisymmetric tube showing segment numbering system.

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APPENDIX A

1. <u>SEGMENT CONTROL CARD</u>

   Format (2I10)

   Columns   1-10   NTYPS (Number of different types of segments;

                            4 maximum)

                  11-20   NTOTS (Number of total segments; 8 maximum)

2. <u>SEGMENT DATA CARDS</u>

   One card for each type of segment.

   Format (F10.5, I10)

   Columns   1-10   THETA (Angle subtended by segment)

                  11-20   NST (The number of segments of each

                            type; 5 maximum)

3. <u>SEGMENT NUMBERING CARDS</u>

   One card for each type of segment.

   Format (5I10)

   Columns   1-10   NUMS (1) (Reference numbers for segments of

                  :   :   each type in

                  41-50   NUMS (5) global numbering system; there can be

                                    one to five numbers per card depending

                                    on how many segments there are of each

                                    type.)

CONNECTING NODES CARDS

One card for each segment. These cards must be in order according to the global numbering system for segments.

Format (8I10)

Columns   1-10   NPC (1) (Nodal number for connecting nodes

                      :   :   according to the axisymmetric

             71-80   NPC (8) grid for the segment).

Cards 5-18 must be repeated for each different type of segment. These are the control cards for the axisymmetric solution.

5. <u>TITLE CARD</u>

       Format (20A4)

       Columns   1-80 TITLE (Title for particular case)

6. <u>CONTROL CARD</u>

       Format (6I5, F5.0, 5I5)

       Columns   1-5   NNLA (Number of nonlinear approximations; NNLA = 1

                             for this version of the program)

              6-10   NUMTC (Number of temperature cards; if -2, a

                             constant temperature is specified)

             11-15   NUMMAT (Number of different materials; 6 maximum)

             16-20   NUMPC (Number of boundary pressure cards; 200 maximum)

             21-25   NUMSC (Number of boundary shear cards; 200 maximum)

             26-30   NUMST (Number of boundary shear cards in tangential

                             direction; 200 maximum)

             31-35   TREF (Reference temperature)

             36-40   INERT (This parameter decides if inertia loads

                             will be present, INERT = 0 means zero values of axial

                             acceleration, and angular acceleration and velocity

                             for each load increment)

	51-55	INCF (If INCF = 0, then surface loads for each time
		increment will be the same as for first increment)
	56-60	IPLOT (Plot parameter, 1 if plot required)

7. MESH GENERATION CONTROL CARD

Format (5I5)

Columns	1-5	MAXI (Maximum value of I in mesh; 25 maximum)
	6-10	MAXJ (Maximum value of J in mesh; 100 maximum)
	11-15	NSEG (Number of line segment cards)
	16-20	NBC (Number of boundary condition cards)
	21-25	NMTL (Number of material block cards)

8. LINE SEGMENT CARDS

The order of line segment cards is immaterial except when plots are requested; in this case, the line segment cards must define the perimeter of the solid continuously. The order of line segment cards defining internal straight lines is always irrelevant.

Format (3(2I3, 2F8.3), I5)

Columns	1-3	I coordinate of 1st point
	4-6	J coordinate of 1st point
	7-14	R coordinate of 1st point
	15-22	Z coordinate of 1st point
	23-25	I coordinate of 2nd point
	26-28	J coordinate of 2nd point
	29-36	R coordinate of 2nd point
	37-44	Z coordinate of 2nd point
	45-47	I coordinate of 3rd point
	48-50	J coordinate of 3rd point
	51-58	R coordinate of 3rd point
	59-66	Z coordinate of 3rd point
	67-71	Line segment type parameter

If the number in column 71 is

0	Point (input only 1st point)
1	straight line (input only 1st and 2nd points)
2	straight line as an internal diagonal (input only 1st and 2nd points)
3	circular arc specified by 1st and 3rd points at the

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ends of the arc and 2nd point at the mid-point
of the arc

4 circular arc specified by 1st and 2nd points at
the ends of the arc with the coordinates of the
center of the arc given as the 3rd point (delete
I and J for 3rd point)

5 straight line as a boundary diagonal for which I
of 1st point is minimum for its row and/or I of
2nd point is minimum for its row (input only 1st
and 2nd points)

6 straight line as a boundary diagonal for which I
of 1st point and/or 2nd point is maximum for its
row (input only 1st and 2nd points)

NOTE: In specifying a circular arc, the points are ordered such that a counter-clockwise direction about the center is obtained upon moving along the boundary.

9. BOUNDARY CONDITION CARDS

Each card assigns a particular boundary condition to a block of elements bounded by I1, I2, J1, J2. For a line I1 = I2 or J1 = J2. For a point I1 = I2 and J1 = J2.

Format (4I5, I10, 3F10.0)

Columns 1-5 Mininum I
    6-10 Maximum I
    11-15 Minimum J
    16-20 Maximum J
    21-30 Boundary condition code
    31-40 Radial boundary condition code, XR
    41-50 Axial boundary condition, XZ
    51-60 Tangential boundary condition XT

If the number in Columns 21-30 is

    XR is the specified R-load and
0   XZ is the specified Z-load and
    XT is the specified T-load

    XR is the specified R-displacement and

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1	XZ is the specified Z-load and
	XT is the specified T-load
	XR is the specified R-load and
2	XZ is the specified Z-displacement and
	XT is the specified T-load
	XR is the specified R-displacement and
3	XZ is the specified Z-displacement and
	XT is the specified T-load
4	XZ is the specified Z-load and
	XT is the specified T-displacement
	XR is the specified R-displacement and
5	XZ is the specified Z-load and
	XT is the specified T-displacement
	XR is the specified R-load and
6	XZ is the specified Z-displacement and
	XT is the specified T-displacement
	XR is the specified R-displacement and
7	XZ is the specified Z-displacement and
	XT is the specified T-displacement

NOTE: All loads are considered to be total forces acting on one radian segment.

10. MATERIAL BLOCK ASSIGNMENT CARD

Each card assigns a material definition number to a block of elements defined by the I, J coordinates.

Format (5I5, 2F10.0, 2I5)

Columns

	1-5	Material definition number (1 through 6)
	6-10	Minimum I
	11-15	Maximum I
	16-20	Minimum J
	21-25	Maximum J
	26-35	Material principal property inclination angle BETA in R-z plane
	36-45	Material principal property inclination angle ALPHA in N-T plane

46-50   IANG (If IANG = 0, then ALPHA is same for total
material block. If IANG = 1, the ALPHA varies
in sign in the I direction from element to
element every NANG elements. This will allow
for equal but opposite helical angles.)

51-55   NANG (Number of elements in the I direction with
the same ALPHA).

11. **PLOT TITLE** CARD*

Format (20A4)

Columns   1-80   Title (Title printed under each plot)

12. PLOT GENERATION INFORMATION CARD*

Format (2F10.0)

Columns   1-10   RMAX (Maximum r coordinate of mesh)

11-20   ZMAX (Maximum z coordinate of mesh)

*NOTE: Use only if IPLOT = 1 (plot required)

13. TEMPERATURE FIELD INFORMATION CARDS

If NUMTC in columns 6-10 of the CONTROL CARD is greater than 1,
the temperature field is given on cards. One card must be supplied for each
point for which a temperature is specified.

Format (3F10.0)

Columns   1-10   R coordinate

11-20   Z coordinate

21-30   Temperature

If NUMTC in columns 6-10 of the CONTROL CARD is -2, a constant temperature field
is specified; the value is given on a single card.

Format (F10.0)

Columns 1-10   Temperature

14. MATERIAL PROPERTY INFORMATION CARDS

The following group of cards must be specified for each material (maximum of 6).

a. MATERIAL IDENTIFICATION CARD

Format (2I5, 2F10.0)

Columns		
	1-5	Material identification number
	6-10	Number of temperatures for which properties are given (12 maximum)
	11-20	Mass density of material (if required)
	21-30	Thermal expansion parameter (If 1, free thermal expansions on the material property cards; otherwise, coefficients of thermal expansion are on the material property cards.)

b. MATERIAL PROPERTY CARDS

Two cards are required for each temperature.

First Card

Format (7F10.0)

Columns		
	1-10	Temperature
	11-20	Modulus of elasticity, E_N
	21-30	Modulus of elasticity, E_S
	31-40	Modulus of elasticity, E_θ
	41-50	Poisson's ratio, ν_{NS}
	51-60	Poisson's ratio, $\nu_{N\theta}$
	61-70	Poisson's ratio, $\nu_{S\theta}$

Second Card

Format (6F10.0)

Columns		
	1-10	Shear Modulus, G_{NS}
	11-20	Shear Modulus, $G_{S\theta}$
	21-30	Shear Modulus, $G_{\theta N}$
	31-40	$\alpha_n T$ or α_n
	41-50	$\alpha_S T$ or α_S
	51-60	$\alpha_T T$ or α_T

15. <u>INERTIA LOAD CARD</u>

Format (3F10.0)

Starting with this input card and including the boundary force cards, this data is to be inputted as a block for each load step, that is NLINC times. There are the following exceptions to this:

a) If INERT = 0, then this card is to be omitted completely (no inertia load).

b) If INCI = 0, then this card is not repeated, but appears in first block only (the inertia loads are constant for each load step).

c) If INCF = 0, then the following boundary pressure and shear cards are to be given only for the first block and not repeated again (the pressure and shear loads are constant for each load increment).

Columns   1-10   ACELZ (axial acceleration

              11-20   ANGVEL (angular velocity)

              21-30   ANGACC (angular acceleration)

16. <u>BOUNDARY PRESSURE CARDS</u>

One card is required for each boundary element which is subjected to a normal pressure, that is the number of these cards is NUMPC for each load increment.

Format (2I5, F10.0)

Columns   1-5   Nodal point M

              6-10   Nodal point N

              11-20   Normal pressure

As shown in the figure below, the boundary element must be on the left when progressing from M to N. Surface normal tension is input as a negative pressure.
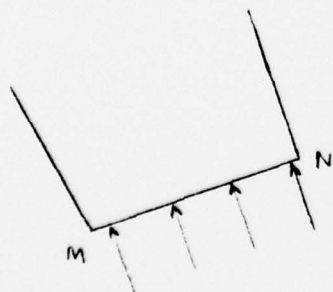
17. BOUNDARY SHEAR CARDS

One card is required for each boundary element which is subjected
to surface shear, that is, the number of these cards is NUMSC for
each load increment.

Format (2I5, F10.0)

Columns   1-5   Nodal point M

6-10   Nodal point N

11-20   Surface shear

As shown in the figure below, the boundary element must be on the
left when progressing from M to N. The positive sense of the
shear is from M to N.

18. BOUNDARY TRANSVERSE SHEAR CARDS
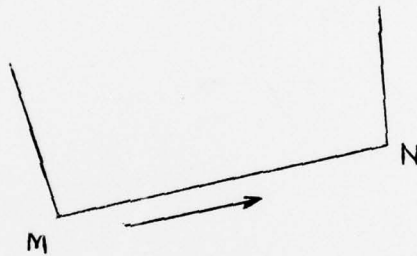
One card is required for each boundary element which is subject
to transverse shear, that is, the number of these cards is
NUMSC for each load increment.

Format (2I5, F10.0)

Columns   1-5   Nodal point M

            6-10   Nodal point N

           11-20   Surface transverse shear
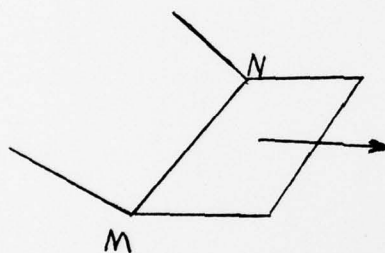
19. BOUNDARY CONDITION CONTROL CARD

       Format (I5)

       Columns 1-5   NRDF (Number of boundary conditions)

20. BOUNDARY CONDITION CARDS

       There are NRDF of these cards.

       Format (I10, F10.0)

       Columns   1-10   NREQ (The number of the equation to be modified

                                in the assembled matrix).

              11-20   U (The actual boundary condition valve).

APPENDIX B

File Number	Useage
1-2	Used in axisymmetric solution
3	Saves product of $[G]^T [S_1]$ for each element from one part of axisymmetric solution for later use.
7-14	These files are used sequentlially for each different segment (i.e. data for segment 1 is stored in file 7; for segment 2 in file 8, etc.). Data saved are {FE}, {FI}, [SK] and $\{u_c\}$, for use in the perturbation analysis
17	Saves the axisymmetric displacement vector and nodal point connections for each type of segment (i.e. data for segments type 1 is stored in file 17 first. Then data for segment type 2 is stored in file 17, etc.).
21-24	Saves [G], [CRZ], and $[BS_1]$ for each element of the segment. Data for each different type of segment goes in a different file. This data is used to calculate stresses and strains when the total displacements are known.
25	Saves $[R_1]^{-1}$ for each different segment in order.

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